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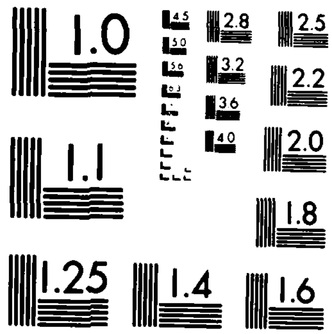

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COMBAT MODELS WITH SPATIAL DEPENDENCE; AN ANALYSIS OF THE MILITARY FACTORS GOVERNING THE EQUATIONS

BY

LIEUTENANT COLONEL DONALD A. LUTZ

15 DECEMBER 1984

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COMBAT MODELS WITH SPATIAL DEPENDENCE;
AN ANALYSIS OF THE MILITARY FACTORS GOVERNING
THE EQUATIONS

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ABSTRACT

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1. Introduction

The concept of combat modeling has never been a particularly well-liked topic in military circles. Clausewitz captured this feeling in describing his abhorrence to the geometric theories of combat prevalent in his day by stating unequivocally[1]:

> In short, absolute, so-called mathematical, factors never find a firm basis in military calculations.

In a sense, Clausewitz's objections are valid and it is presumptuous to believe that some mathematical formulas can duplicate the tremendously complex nature of combat, even to hope to approximate all the non-quantifiable human factors that enter the situation, or take into account all the random phenomena. On the other hand, his advice has been ignored in recent decades by combat model analysts who expend considerable resources on mathematical modeling and have a significant influence on military planning both in the United States and the Soviet Union. This dichotomy comes about because while no one seriously believes that combat models are a real substitute for combat or combat experience, they are, for many purposes, preferable to a complete lack of knowledge or even more unacceptable alternatives. They can be useful, for example, at a strategic level for making projections or net-assessments, for determining the possible effects of an increase or decrease of troop levels, or for measuring the change in the situation caused by the introduction of a new weapons system having a dramatic increase in combat effectiveness. At the tactical level, combat models may be used in the planning process to provide a quantitative (or rational) basis for deciding between alternative courses of action. This type of application is more common for the Soviets, who often make claims of the following kind:

> (The) use of ... models ... makes it possible to forecast the course and outcome of the combat operations in question. If this forcast is unfavorable, it will be necessary to take the

> appropriate measure (change the initial correlation of forces, the tactics of employment of certain weapons, the plan for bringing up the reserves, etc.) so that the future real combat operations proceed in the required direction.[2]

While Clausewitz on one hand rejects using deterministic mathematical formulas to predict the outcome of combat, he also provides a good example showing why such methods are sometimes necessary. In discussing several courses of action for the timely employment of reserve forces,[3] it becomes clear from his tentative remarks that what he really needs is a quantitative measure for the comparison. Without such a measure, he is forced to rationalize a decision based on experience. That can be appropriate in certain unchanging or very similar situations, but in others it can lead to far from optimal solutions. Clausewitz describes the decisions facing a commander as resembling "mathematical problems worthy of the gifts of a Newton or an Euler."[4] In his era the scientific support needed to crack such problems was not available. In our age it is.

The point of view taken in this paper is not to try to justify the use of combat models, but to recognize that since they exist and are often used, operations planners should at least try to understand what basic assumptions go into a model and thereby be able to discuss their advantages or disadvantages and their relevance or unrelevance in a particular situation. This point of view is frequently overlooked. Advocates of combat modeling are often so enthralled by computers spewing out data that they don't care, or forget, what factors the computer is using to generate that data. Detractors point to the ludicracy of believing that the numbers contain any semblance of reality and dismiss them without understanding what factors are being modeled and how the results should be interpreted. Without understanding, analyzing, critiquing, modifying, and carefully monitoring these factors and noting their influence on the calculations,

it is pointless to use combat models or believe in their output. Moreover, this type of critical analysis is essential for a more in depth understanding of the combat process and how it is evolving, whether or not the validity of any particular model is accepted.

Most models that simulate the combat process involve in some manner two vital aspects of combat: attrition (combat losses) and manuever (movement of forces on the battlefield during combat). Usually the calculations for these quantities (combat losses incurred and distance traveled by an attacking unit during a specified time interval of combat) are made independently in two sub-models of the process. The first, an attrition model, treats combat as a static (with respect to motion) problem and calculates combat losses based on certain assumptions involving enemy forces, friendly forces, and their relative combat potency. The second, a manuever model, usually disregards attrition during movement and uses past results of the attrition model to calculate how far the attacking forces should have advanced during the combat period, based on factors such as mobility, terrain, and the preponderance of attacking forces to defending forces during the combat period. Thus a combat operation is decomposed into a sequence of engagements of fixed time duration and in each such engagement the results of combat are calculated in terms of losses (friendly and enemy) and gains (terrain objectives). To model a larger scale combat process consisting of various forces distributed over a two-dimensional battlefield, it is also common to partition the battlefield into geometric regions of various sizes/shapes (rectangles or hexagons, for example) and run the sub-models separately for forces in adjacent regions. All of this becomes a sizable management problem. But using computers to store data, make calculations, and monitor the sequence of events, it represents a reasonable and feasible approach to combat modeling.

Other features involved in a combat operation can also be included without

greatly complexifying the calculations. These may include logistical considerations, air battles, air supremacy, and tactical air support, or any number of other combat support or combat service support activities that can be quantified. But for the remainder of this paper, such auxiliary or ancillary features will not be discussed and the focus will just concern the principal elements of ground combat: attrition, manuever, and the spatial distribution of forces.

Another way of modeling a combat operation taking place in a given region and specified time period would be to attempt to include all three principal features together in a single module and take into account their strong interdepencies on a continuous rather than a discrete basis. For example, high attrition during the assault phase will greatly influence the depth of penetration. Pockets of resistance may be bypassed and the avenues of least resistance may be followed. Exploitation may occur earlier than expected due to overwhelming defender losses.

The United States mainly follows the first route indicated above. While our combat models in many cases have evolved into tremendously complex edifaces, encompassing a great many features of combat and demanding huge date requirements and computer/man hours to prepare and run one scenario, the basic combat quantities (attrition, manuever, spatial distribution of forces) are treated in a relatively simple and disjoint manner and interactions only occur at discrete times during the course of modeling an operation.

The Soviets, on the other hand, seem to consider both routes in combat modeling.

> "A model will be the most adequate when a third factor, the spatial factor, can be considered. The importance of this is difficult to over estimate."[5]

During the past 15 years some models have been proposed[6] in Soviet military operations research writings that integrate all three of the principal combat features

on an interactive basis and continuously in space and time. These models (on the surface) indicate a capability and sophistication far beyond that displayed in United States combat modeling techniques.

The goal of this paper is to perform a critical analysis of the factors entering into some equations that the Soviets have considered and to identify assumptions used in their derivation and implications that these assumptions may have relative to the way the United States operational planners view both United States and Soviet combat modeling processes. This represents just the first step in understanding these more complex models. The questions of data requirements, feasibility of the calculations, sensitivity of the model, and direct comparison of the results (attrition, manuever) with United States models are all beyond the scope of this paper. They would also require more detailed information about certain unspecified factors in the equations than are presently available.

2. Combat Modeling Equations

Combat equations are nothing more than expressions that tell how certain combat parameters are changing with respect to time and/or location on the battlefield. It is, in many cases, much easier to describe the change that takes place in a quantity and what affects that change than it is to give an explicit formula for the quantity itself. This is the main reason how and why differential and difference equations are so useful in modeling processes in the physical or social sciences. The equations that arise may not even be explicitly solvable (in terms of a functional representation) but frequently they can nevertheless be analyzed and solved numerically.

The simplest combat equations satisfying this description were first considered by F. Lanchester[7] (although some Soviets[8] claim "they were put forward by Osipov earlier"). Lanchester's equations form the basis of most treatments of the combat attrition process, whether it is to be considered independently from the process of manuever or not. They are also the easiest place to begin a discussion of combat dynamics and will provide a good introduction to the more complex case to be treated later.

In Lanchester's setting, two forces engage in combat over a period of time but their specific terrain location, their geographical relationship to each other, and their schemes of manuever are not considered. The only changing quantities are the number of combat elements (measured in either individual troops, weapons, or standardized units) and in the simplest cases they are even assumed to be homogeneous, that is, all the elements are considered as identical or interchangeable. Two cases (sometimes called Model A and Model B, or the Linear Law and the Square Law) are distinguished, depending upon whether the combat situation is "organized" or "unorganized."

In the first case of organized combat, the forces are treated as idealized

"point forces," where all fire is assumed to be aimed and shifted when a target is hit (this means perfect surveillance), and every target on one side is capable of being hit by each weapon on the opposing side. If $N(t)$ denotes the number of friendly combat elements at time t, then according to these assumptions their loss rate is proportional to $N_e(t)$, the number of enemy forces firing on them, with a constant of proportionality c_e that in some sense describes the enemy's combat effectiveness. Similarly, the change in enemy strength is equal to the product $cN(t)$, where c is the combat effectiveness coefficient for the friendly forces. Putting these together gives rise to the equations,

$$\frac{dN(t)}{dt} = -c_e N_e(t) \quad \text{and} \quad \frac{dN_e(t)}{dt} = -cN(t),$$

where the expressions on the left hand side (called derivatives) signify the instantaneous rate of change in the numerical strengths with respect to time. The minus sign indicates the forces are being attrited and no operational losses (ones not due to enemy action) or reserve forces are being considered. It is not the intent here to discuss more exactly what the constants c and c_e represent, how they could be calculated, or how to solve these equations. (For a very readable treatment, however, see Taylor[9].)

In the second case of unorganized combat, each force is assumed to be homogeneous and spread evenly on an area basis, but again the spatial relationship between the forces and their movement are not considered. All fire is assumed to be evenly distributed on an area basis (such as artillery, for example) and fire is not necessarily shifted when a target is hit. The loss rate for each force is then proportional to the product of its own and the enemy's strength, since the probability of a hit depends upon the density of the friendly forces in the area and how many enemy weapons are firing into it. The constants of proportionality a, a_e in this case depend upon the lethality of

the weapons (on an area basis) and their effective rate of fire. In the same way as above, this gives rise to the equations

$$\frac{dN(t)}{dt} = -a_e N(t) N_e(t) \quad \text{and} \quad \frac{dN_e(t)}{dt} = -aN(t)N_e(t).$$

Even though in this case there is some reference to an area over which the forces are distributed, this is also, in effect, a spatially-independent model since the forces are not assumed to be moving or dispersing (other than by attrition) and the areas are assumed to be fixed.

Since combat loss can more realistically be attributed to both direct (aimed) and indirect (unaimed) fire, a somewhat more convincing spatially-independent model can be constructed by combining the two systems of equations, using a factor that accounts for the dispersion of forces on the battlefield.[10]

When spatial dependence and manuever is introduced into the attrition process, the model can become substantially more complex. Whether the forces are considered to be distributed along a line or curve (one-dimensional) or spread over a two-dimensional region is not much different in concept, but the data requirements and calculations could prove to be enormously more difficult. (J. Taylor [8] has discussed the one-dimensional case and his arguments are extended here to the two-dimensional setting.)

The combat process in the large can be likened in several ways to the following problem, which has several similarities with standard fluid problems, but also some significant differences: Imagine two fluids are placed in a flat region (the depth is not essential), let them be separated by a thin, flexible, and stretchable membrane (so that they don't mix), and let the fluids have different densities in different parts of the region. Suppose now that one fluid "advances toward" the other by specifying a direction of motion in a certain sector of the region and an initial velocity. Further-

more, assume that elements of the fluids that are close enough together can annihilate each other at a rate determined in some way by the fluid densities and the "combat effectiveness" of the fluid elements. The problem is to calculate how far the attacking fluid will advance toward the defender and how much of both fluids are left after a certain interval of combat.

There are two major differences between this problem and a simple fluid dynamics problem. The principal distinguishing feature of combat is attrition, which is usually not present in nature: matter can neither be created nor destroyed. So the usual conservation of mass (matter) equations will have to be modified. Another feature commonly assumed in simple fluid problems is that the fluids are homogeneous, whereas it is typical in combat dynamics to have forces with either a very high degree of concentration or dispersion, especially in an assault. To quantify this property, density functions are introduced to take place of the counting functions $N(t)$, $N_e(t)$ in a pure attrition model. In physical problems density is usually thought of as mass per unit area. This interpretation also carries over to combat problems, where "mass" can either mean the raw number of combat elements (troops or weapons systems) or else these raw numbers can be weighted to indicate a greater or lesser combat power from some established norm.

Using the analogy between combat and fluid dynamics, one way to derive equations for a combat dynamics model is to begin with the standard equations for fluids in motion[11] and modify them by introducing factors corresponding to the attrition and augmentation of forces during combat. This approach has been indicated by some Soviet military writers. But it is somewhat more instructive and informative to derive the equations from more elementary reasoning and see how the terms that enter the equations reflect the basic assumptions that are made.

To do this, consider a two-dimensional region (Figure 1) with grid coordinates (x,y), where x and y measure distances along two perpendicular axes (say x corresponds to the east-west direction and y to the north-south). At each instant of time t and each position (x,y) on the battlefield, let $\rho(t,x,y)$ denote the density of friendly forces. For instance, suppose a 4000-man brigade were equally dispersed over a certain square kilometer area. Then the density of forces at each such point would be $4000/km^2 = 2/5$ per 10-meter square. (Note the density does not have to be a whole number.) To find the number of troops in a region of size $500m^2$, multiply 2/5 by 5 (the number of 10 meter squares) to get 2 men per $500m^2$. We also let $\rho_e(t,x,y)$ denote the density of enemy forces. If we assume that the forces are in contact, but separated, then for each fixed time t there will be an idealized curve (called the FEBA) where $\rho(t,x,y) = 0$ for all points (x,y) on one side while $\rho_e(t,x,y) = 0$ for all points on the other side. If the combat is also restricted by lateral boundaries, then the densities would be zero on these boundaries (and outside the region as well) for all time t.

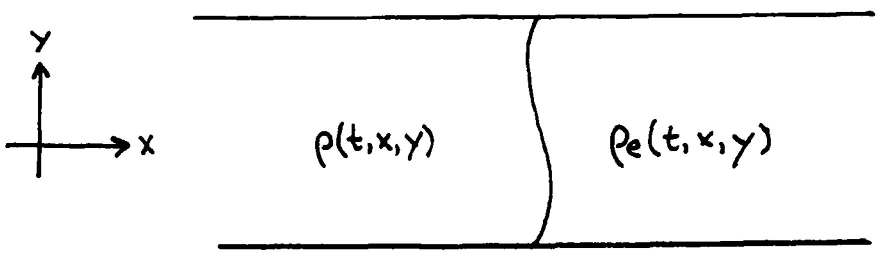

Figure 1

Quantities also need to be introduced that measure the rate of movement (velocity) of friendly and enemy forces on the battlefield. Since this is a two-dimensional problem, the velocity will have two components, denoted $u(t,x,y)$ and $v(t,x,y)$. The first one gives the velocity component in the horizontal (east-west) direction, where a positive quantity will mean the

flow is from west to east. Similarly, $v(t,x,y)$ measures the velocity component in the vertical (north-south) direction, with a positive velocity indicating motion from south to north. (In most combat situations with a battlefield having horizontal lateral boundaries (as in Figure 1), the horizontal component will be the more significant one. Only in turning movements or envelopments will the vertical component take on much importance.)

The combat process is characterized by two types of equations, which are directly attributed to certain conservation principles. The first, termed <u>conservation of mass</u> represents a balance between the number of combat elements passing through an area, the number that are attrited due to enemy action, and the number that are introduced as reserves. The second, called <u>conservation of momentum</u> represents a balance between the rate of movement of the combat elements and any forces that inhibit or enhance that motion. Typical of these forces are defender resistance, pressure of the attack forcing the defender to withdraw, and externally induced command and control measures. The resistance mainly depends on the preponderance of forces in a sector of the battlefield; the closer this ratio is to one, the lesser the momentum of the attack.

To derive the form of the conservation of mass equation, imagine a small rectangle in the battle area with dimensions Δx and Δy. (Think of Δx and Δy as increments on the x and y directions, respectively, and especially as relatively small quantities.) If (x,y) are the coordinates of the lower left hand corner of the rectangle (as in Figure 2),

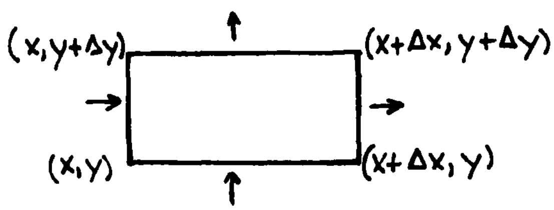

Figure 2

then the change in the number of friendly elements in the rectangle from time t to $t + \Delta t$ equals the number of friendly forces entering the rectangle minus the sum of the number (of survivors) leaving it and the number of casualities that occur within it. The number of elements in the region at any time is the density times the area $\Delta x \Delta y$, so the change in this number can be expressed as

$$\rho(t+\Delta t,x,y)\ \Delta x \Delta y - \rho(t,x,y)\ \Delta x \Delta y.$$

To determine the number of friendly forces entering the region, recall that we think of the positive x-direction in the flow as being from west to east and the positive y-direction from south to north. Then the number of elements entering on the left hand border of the rectangle during the time interval Δt is the velocity of the elements along the x-direction times their number times Δt, or

$$u(t,x,y)\ \rho(t,x,y)\ \Delta y \Delta t.$$

Actually, this is just a good approximation to the number of elements entering the rectangle since the values $u(t,x,y)$ and $\rho(t,x,y)$ are not the same at all points on the left border, but depend upon a value between y and $y + \Delta y$. However, when Δy is small this difference is also small in comparison with the main terms and will not affect the final result. Similarly, the number of forces leaving along the right hand border is given by

$$u(t,x+\Delta x,y)\ \rho(t,x+\Delta x,y)\ \Delta y \Delta t,$$

the number entering from the bottom is

$$v(t,x,y)\ \rho(t,x,y)\ \Delta x \Delta t,$$

and the number leaving from the top is

$$v(t,x,y+\Delta y)\ \rho(t,x,y+\Delta y)\ \Delta x \Delta t.$$

If the signs of the velocity components would be negative, it just means the flow is taking place in the opposite direction and the above terms are still correct. The terms introduced above describe the combat elements moving in and out of the rectangle without regard to combat. Other changes in the

density take place due to attrition and reinforcement. The latter is easy to introduce because the reinforcement rate is independent of the other quantities in the problem. It is just necessary to construct a function (of t,x, and y) that equals the rate at which the density is increased for particular times t and positions (x,y). The attrition rate is more complicated to determine because it is strongly dependent on the densities of the opposing sides (as in the Lanchester cases). The model described by Chuyev and Mikhailov[12] does not explicitly present a formula for the attrition, which is one of the significant intelligence gaps in the model and one that could seriously impact on its usefulness and credibility. Taylor[13] has discussed a general procedure for constructing an attrition function based on Lanchester-type arguments. More will be said about this in the next section. We will just use the expressions $f(\rho,\rho_e)$ and $f_e(\rho,\rho_e)$ to denote the attrition rates in the densities due to friendly and enemy fire, with the notation indicating (as in Lanchester's cases) that they may depend upon both the densities of friendly and enemy forces.

If we first equate

$$[\rho(t+\Delta x,x,y) - \rho(t,x,y)]\ \Delta x\Delta y$$

with the terms

$$[u(t,x,y)\ \rho(t,x,y) - u(t,x+\Delta x,y)\ \rho(t,x+\Delta x,y)]\ \Delta y\Delta t$$

$$+\ [v(t,x,y)\ \rho(t,x,y) - v(t,x,y+\Delta y)\ \rho(t,x,y+\Delta y)]\ \Delta x\Delta t,$$

subtracting off $f_e(\rho,\rho_e)\ \Delta x\Delta y$ (the number of friendly casualties in the rectangle) and adding in the number of replacements, and then divide by $\Delta x\Delta y\Delta t$ and let all the quantities Δt, Δx, Δy approach 0, the equation

$$(1)\quad \frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + \rho\frac{\partial u}{\partial x} + v\frac{\partial\rho}{\partial y} + \rho\frac{\partial v}{\partial y} = -f_e(\rho,\rho e) + r$$

is derived, where r denotes the replacement rate (converted to a density). All the quantities ρ, ρ_e, u, v, and r depend upon t, x, and y, but that notation is suppressed in order to make the equation more compact. The expressions involving the symbol ∂ are called partial derivatives and indicate the instantaneous rate of change only with respect to one of the variables t, x, or y. For example, $\partial\rho/\partial t$ describes the change in the density only when t changes, while $\partial\rho/\partial x$ describes the change when x changes, but the other two variables t and y are fixed. The equation (1) above is called a partial differential equation because it contains partial derivatives, as compared with the Lanchester equations which are known as ordinary differential equations. There is also a corresponding conservation of mass equation for the enemy forces which is obtained by replacing ρ by ρ_e, u and v by u_e and v_e (the velocity components of the enemy force), f_e by f, and r by r_e, the replacement rate for enemy forces.

The conservation of mass equations tell nothing about how to calculate the velocities u and v of the attacking force. Experience indicates that the velocity of an attack is closely related to the relative advantage the attacker has over the defender in a particular sector of the battlefield. The quantification of this relationship gives rise to conservation of momentum equations. In the physical sciences (say in fluid flow) momentum is defined to be mass times velocity. The same concept fits the combat situation. The equations come about by balancing all the factors that can contribute to changes in the velocities in the x and y directions. There are two principal reasons for changes, an internal cause (called resistance) due to a loss or gain in relative combat power and external causes due to command and control decisions (slowing or speeding the attack or changing its direction). The relative combat power is what the Soviets refer to as the "correlation of

forces" and represents a ratio of total combat power of the attacker to that of the defender. If the combat effectiveness coefficients are equal, this equals the ratio of the numbers of combat elements, or just the ratio of the densities of the forces. So to model the effect of a change in velocity of an attack, it is necessary to construct an expression that depends upon the ratio of densities $\rho/\rho e$ and also v_{max}, the maximum possible velocity the attackers could achieve (for specified terrain, weather conditions, and intrinsic transportation assets) if the combat were unopposed. The Soviet model expresses this quantity as

$$F = v_{max} \frac{4(\frac{\rho}{\rho e}) \frac{d}{dt} (\frac{\rho}{\rho e})}{\left[(\frac{\rho}{\rho e})^2 + 1 \right]^2} = -2 v_{max} \frac{d}{dt} [1 + (\frac{\rho}{\rho e})^2]^{-1},$$

but does not explain how it was derived.

If we now consider the horizontal velocity component $u = u(t,x,y)$ along a trajectory $x = x(t)$, $y = y(t)$ in the combat region, then the change in u with respect to t comes about due to the explicit dependence of u upon t and also the implicit dependence via x and y. Using the so called "chain rule" to calculate this rate of change of u with respect to t (and the fact that $u = dx/dt$ and $v = dy/dt$) as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y},$$

and setting this equal to $-F_x + \gamma$, where F_x is the component of F (above) in the x-direction and γ is a control parameter, we derive

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -F_x + \gamma \tag{2}$$

as one of the conservation of momentum equations. Another similar equation is created by considering the change in v with respect to t and equating that to $-F_y$, the y-component of F plus y-controller term. For the velocity components of the enemy force, two analogous equations should also be introduced. For this purpose, it is necessary to construct an expression somewhat similar to the F above that expresses the dependence of the withdrawal rate of enemy forces induced by the pressure of the attacking force. This should also depend upon the correlation of forces ρ/ρ_e, but the exact functional dependence and its dependence on v_{max} for the defenders is not obvious. Since the Soviet model does not include such a term, no more needs to be said about it at this time.

In summary, a model of combat dynamics in a (two-dimensional) spatial setting may involve 6 partial differential equations: two conservation of mass equations and four conservation of momentum equations. Additionally, some <u>starting conditions</u> need to be stated that correspond to initial troop dispositions and an initial velocity for the attacking force (say at time $t = 0$). Finally, some <u>interface conditions</u> need to be identified for the densities ρ and ρ_e that correspond to the property that the forces are always in contact. What extra assumptions and conditions need to be imposed to be able to call this a well-posed problem will not be discussed here. Instead, the focus will now shift from these equations to similar ones proposed by Chuyev and Mikhaylov. We will discuss the differences and what this might mean for the Soviet model.

3. A Soviet Spatially-Dependent Combat Model

The model described in Section 2 was derived under some fairly general assumptions on the combat dynamics. The basic forms of the terms on the left hand sides of the equations would be common for any two-dimensional flow problem. The terms on the right hand sides are the important ones from the aspect of identifying other assumptions in the model.

As remarked earlier, there is insufficient information about the attrition rates $f(\rho,\rho_e)$ and $f_e(\rho,\rho_e)$. It is highly likely, however, that given the Soviet's frequent usage of Lanchester equations in the non-spatially dependent setting, these rates will also be based on Lanchestrian considerations. Moreover, since the notation indicates that they depend upon both ρ and ρ_e, some combination of Models A and B is indicated. One further modification of Lanchester's attrition rates is called for in a spatially-dependent model. Recall that in the basic Lanchester models all friendly targets are assumed to be within range of every enemy weapon and visa versa. For the spatial model it would make sense to weigh the attrition rates by considering only those enemy weapons in range to fire on targets. Taylor [8] has discussed such factors.

The essential term on the right hand side of the conservation of momentum equations is the force F describing resistance to movement. Here, the Soviets have given an indication of a form they consider feasible, although from the tone of the statement it is difficult to determine whether the same formula is used in all cases. More will be said about the implications of this particular F in what follows.

The most striking feature of the Chuyev-Mikhailov equations (hence forth called the Soviet Model), however, as compared with the general model discussed in Section 2 is that there are only four equations, not six. This has some significant implications that may point to weaknesses and limitations in its

usefulness.

The Soviet Model contains two conservation of mass equations having forms exactly as given above, but only two conservation of momentum equations. The reason that two equations suffice is that the velocities u and u_e and v and v_e have been equated and the common values are identified with the rates of movement of the FEBA (in the horizontal and vertical directions). While at first glance this does not seem to have much special significance, when compared with the derivation of the conservation of mass equations, this assumption is seen to have some strong consequences. Go back to the rectangle in Figure 2 and recall how the velocity components entered into that discussion. Assuming that $u = u(t,x,y)$ represents the rate of movement of the FEBA (in the x-direction) and using this u to calculate the number of forces entering the rectangle is tantamount to assuming that no compression or dispersion of forces takes place during combat; all forces maintain their interval and advance or fall back at a rate strictly determined by the rate of movement of the FEBA. It follows that this rate of advance u in effect only depends upon time and possibly on the vertical component y, but probably not on x in any essential way.

While from the aspect of combat modeling this is certainly an admissible assumption and the model still represents a substantial advance over the simple Lanchester models, the resulting model nevertheless has some serious drawbacks and does not have the same potential as a 6 equation model. For example, in most assaults one imagines that the defenders maintain positions for a certain period after the initiation of combat while the attacking force is concentrating along the FEBA. The FEBA begins moving rather slowly compared to the rate of advance of the attackers. The difference between the rate of advance and the rate of withdrawal is accounted for in the concentration of forces for the attack. The pressure created by this concentration only begins to force the

defenders to withdraw after the defenders experience sufficient losses and the superiority of the attacking force has been realized. This indicates that the Soviet Model may forecast a substantially larger depth of penetration than is realistic, or is predicted using four conservation of momentum equations.

Another feature of the Soviet Model that reflects a larger than reasonable advantage to the attacker can be inferred from the particular form for the resistance force F described in Section 2. Using the fact that along a trajectory the left hand side of (2) just represents du/dt, the rate of change in the movement in the x-direction (of the FEBA), then (2) becomes

$$\frac{du}{dt} = -v_{max} \frac{d}{dt} \frac{2}{(\rho/\rho_e)^2 + 1},$$

and integrating this one can conclude that

$$u = v_{max} \frac{(\rho/\rho_e)^2 - 1}{(\rho/\rho_e)^2 + 1}.$$

Here, v_{max} denotes the component of the maximum velocity in the x-direction of the attacking force if the movement were completely unopposed. For a correlation forces ratio of 2:1, this predicts that the FEBA will move at a rate of 3/4 of v_{max}. For a ratio of 3:1, the FEBA would move at 15/16 times v_{max}. This yields an unrealistically large rate of advance compared to combat experience. Formulas of this type, however, have been observed in other Soviet military writings.

A final limitation of the Soviet Model as compared with the general six equation model concerns its potential usage. Because the rate of advance of the FEBA is assumed to represent both the velocity of the attacker and defender, it is not possible to model courses of action where the defender would mount limited counter attacks. Such situations implicitly violate the assumptions in the construction of the Soviet Model.

4. Comparison with United States Models

As mentioned earlier, United States Combat Models[14] tend to be synthesized by sequentially employing separate sub-models where, in particular, attrition and movement are handled disjointly. While some models may reflect a low-grade, constant rate of attrition during movement, the basic philosophy contends that movement and attrition can, for all practical purposes, be effectively separated. Some United States operational researchers[15] have from time to time considered more comprehensive combined function models, but no model presently in the United States catalog uses an approach similar to the partial differential equation model derived in Section 2 or the Soviet modification described in Section 3.

This does not necessarily imply that the results of combat modeling using the United States and Soviet approaches are much different, only that their theoretical bases are apparently different. Often it can happen that models that appear to differ greatly actually produce results that are close because one model is just a discrete version of the other, formed by breaking the time and space variables into disjoint steps. If the step sizes are small enough (in a way that can be made precise after a numerical analysis of the problem), then the discrete version will usually agree fairly well with the continuous version. But the discrete version also needs to accurately reflect the linking mechanism from the continuous case.

In a discrete model, the linking process often involves both internal (program) rules and external (player) decisions of how results of combat during the separate time intervals and spatial regions (hexagons or rectangles) is to be combined (or integrated). Such rules or decisions may involve whether to continue the attack, change its direction or intensity, introduce reinforcements, or (for the defender) when to withdraw and at what rate. A partial differential equation model contains an internal mechanism for making these decisions (except

reinforcing) on a continuous basis and could be considered as self-driving or free-standing. Whether a continuous and a discrete model produce results that are close or not depend on how closely the rules agree. In a discrete model it is common to break off an attack when the strength of the attacking force in a region is reduced below a specified level. In the partial differential equation models, combat will automatically continue between contiguous units and the FEBA will move as long as there is a favorable correlation of forces ratio for the attacker. There is no way of telling, for example, whether a region of a certain fixed size is being uniformly occupied by a battalion at 90% strength or a regiment at 30% strength. The density will be the same.

Two properties of the Soviet Model described in Section 3 already have indicated an inflated estimate for depth of penetration in an attack. Another factor contributing to this comes from a comparison of the self-driving feature of the Soviet Model with any of the discrete models. The contention here is that with a discrete model there is more of a tendency to break off an attack when an attacking unit in a particular region (hexagon or rectangle) of the battlefield becomes degraded beyond a designated level (that guarantees unit cohesiveness). If this degradation occurs uniformly over the entire sector, the attack will necessarily have to be discontinued, at least until reinforcing units can be brought forward or suitable augmentations or regroupings can be made in the attacking forces. In a continuous model, there is no intrinsic provision for this. The attack will continue and the forces will advance as long as the attackers maintain a favorable correlation of forces, even though the individual units may be degraded far beyond their capability to sustain combat. While the Soviet Model could, in principle, be modified to avoid this unrealistic feature, there is no indication that the Soviets do this or even would consider it advisable or necessary.

The self-driving property of the Soviet Model appears on the surface at least to be inconsistent with normal Soviet command and control practices, which involve traditionally a high degree of centralization. The Soviet Model implies highly decentralized control procedures, where individual unit commanders would be given great freedom and flexibility in directing an attack and expected to take advantage of all opportunities to exploit the situation. This inconsistency is understandable from the Soviet perspective and characteristic of many of its planning processes. The assumptions made by the planners at higher headquarters is usually not matched by the delegation of authority to actually perform as expected. This shortfall is endemic of Soviet planning, especially in economic management. The use of this model shows how the same phenomenon may arise in the military sector.

The only feature of the equations that could materially moderate the inflated estimates produced by the Soviet Model involves the calculation of the attrition factors that enter into the conservation of mass equations. Both the general form of these rates (whether to use Lanchester A, B, or a combination) and the values of the combat potential scores could have a substantial impact on the outcome. Once the general form is established, it will be possible to see how sensitive the outcome is to changes in combat potential scores and whether (and to what degree) the use of these equations is creating false expectations on the part of Soviet planners at higher levels.

5. Conclusions

On the surface, the Soviet (partial differential equation) Model looks like a vast theoretical improvement over the simpler models based on the elementary Lanchester equations. How much of this is an illusion and how much is substantial still remains to be seen. (One major difficulty is caused by the theoretical complexity of the equations: they cannot, in general, be explicity solved as in the Lanchester case.) Much more work needs to be done before the model and all its ramifications can be fully understood. The model has several inherent advantages as well as some peculiar liabilities.

One outstanding feature of the model is its self-driving capability. It turns out to be both an advantage and a disadvantage. Two advantages are that the model can accept as input the same type of scenario for a proposed course of action (initial dispositions and axes of advance for the main and supporting attacks) that would be given by command guidance and process it in a way that is consistent with some of the standard principles of combat. A disadvantage is that this same self-driving capability, if not restricted, can lead to unrealistic combat situations and overly optimistic expectations. Overall, the evidence indicates an exaggerated rate of advance for an attacking force as compared with United States models, but a precise comparison has not yet been made. Some of the features (discussed earlier) that contribute to this property of the model could be modified within the present structure of the equations; others are more intrinsically part of the system.

A final determination of the advantages, disadvantages, strengths, and weaknesses of the model can only be made after certain requirements for additional information are met. These fall into the following two broad classes:

Intelligence Requirements

(1) Determine the functional expressions for the attrition factors used in the conservation of mass equations.

(2) Determine the fixed data requirements for the model, specifically the combat efficiency parameters that enter the attrition factors and how they can be calculated.

Analytical Requirements

(1) Determine what constitutes a well-posed problem for the model by specifying the precise degree of freedom in the initial conditions, discussing the existence and uniqueness of solutions, and finding the sensitivity of the output to changes in the initial data.

(2) Determine feasible numerical procedures to solve the equations along with an estimate of necessary computer power.

(3) Compare the results of the Soviet Model with some of the discrete United States models.

(4) Analyze the degree of dependence of the output on the fixed imput data, specifically the combat effectiveness parameters.

(5) Determine the advantages or disadvantages caused by the assumption (in the Soviet Model) that the rate of movement of the FEBA equals the rate of movements of all components of the two opposing forces.

Donald A. Lutz

FOOTNOTES

1. Carl von Clausewitz, On War, p. 86.

2. Yu. V. Chuyev and Yu. B. Mikhaylov, Forecasting in Military Affairs, p. 106.

3. Clausewitz, p. 205.

4. Clausewitz, p. 112.

5. K. V. Tarakanov, Mathematics and Armed Combat, p. 206.

6. Chuyev and Mikhaylov, p. 105.

7. James G. Taylor, Lanchester Models of Warfare, vol. I, p. 52.

8. Chuyev and Mikhaylov, p. 104.

9. Taylor, pp. 52-102.

10. Ye. S. Venttsel', Introduction to Operations Research, pp. 234-240.

11. E. Zauderer, Partial Differential Equations of Applied Mathematics, p. 514.

12. Chuyev and Mikhaylov, p. 105.

13. James G. Taylor, Lanchester Models that reflect continuous spatial distribution of forces, pp. 7-8.

14. See [3], [4], and [11].

15. Koopman, Analytical Treatment of a War Game, pp. 727-735.

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[10] E. Zauderer, Partial Differential Equations of Applied Mathematics, J. Wiley - Interscience, New York, 1983.

[11] Player's Guide to the USAWC Theater and Corps Operations and Planning Simulation, TACOPS (MTM), Carlisle Barracks, PA, 1984.

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